Name: AP 2000 A II | Klasse | Datum | Seite | Blatt

1.1 Sym: $f(-x) = -2x + 2x\,e^{1-(-x)^2} = -\left(2x - 2x\,e^{1-x^2}\right) = -f(x)$

Also: $f(-x) = -f(x) \Rightarrow$ P. Sym. zum Ursprung

NST: $f(x) = 2x\left(1 - e^{1-x^2}\right)$ (← Faktorisieren!) ; $\underline{x_1 = 0}$

$1 - e^{1-x^2} = 0 \Leftrightarrow e^{1-x^2} = 1 \quad /\ln \; ; \; \ln(1) = 0$

$1 - x^2 = 0 \Leftrightarrow x^2 = 1 \Rightarrow \underline{x_2 = -1} \; ; \; \underline{x_3 = 1}$ (Sym!)

1.2 Für $x \to \infty$: $2x\,e^{1-x^2} \to$ "$\infty \cdot 0$" : Bruch für L.H. herstellen!

$$f(x) = \frac{2x}{e^{-(1-x^2)}} = \frac{\overset{\to\infty}{2x}}{\underbrace{e^{x^2-1}}_{\to\infty}} \xrightarrow{L.H} \frac{2}{\underbrace{2x\cdot e^{x^2-1}}_{\to\infty\cdot\infty}} \to \underline{0}$$

$\underline{y = 2x}$ ist eine $\underline{\text{schräge Asymptote}}$

1.3 $f(x) = 2x\left(1 - e^{1-x^2}\right)$; P-Regel ; $\left(1 - e^{1-x^2}\right)' = 2x\,e^{1-x^2}$

$f'(x) = 2x\left(2x\,e^{1-x^2}\right) + 2\left(1 - e^{1-x^2}\right) = \underline{2 - 2e^{1-x^2} + \underbrace{4x^2 e^{1-x^2}}}$

$f''(x) = 0 \overset{-2\cdot(-2x)}{+ 2\cdot 2x}\,e^{1-x^2} + 8x\,e^{1-x^2} + 4x^2\cdot(-2x)\,e^{1-x^2}$ P-Regel!

$= \left(4x + 8x - 8x^3\right)e^{1-x^2} = \underline{4x\left(3 - 2x^2\right)e^{1-x^2}}$

1.4 $f''(x) = 4x\left(3 - 2x^2\right)\underbrace{e^{1-x^2}}_{>0} = 0 \Rightarrow 4x\left(3-2x^2\right) = 0$; $x_1 = 0$

$3 - 2x^2 = 0 \Leftrightarrow x^2 = \frac{3}{2} \Rightarrow x_2 = -\sqrt{\frac{3}{2}} = -\frac{1}{2}\sqrt{6}$; $x_3 = \frac{1}{2}\sqrt{6}$

Alle NST sind 1-f m. VZW ; $f''(x) = \left(-8x^3 + 12x\right)\underbrace{e^{1-x^2}}_{>0}$

x		$-\frac{1}{2}\sqrt{6}$		0		$\frac{1}{2}\sqrt{6}$	
VZ $f''(x)$	$+$	0	$-$	0	$+$	0	$-$
G_f	rekr	W_1	likr	W_2	rekr.	W_3	likr

; Intervalle nicht explizit verlangt

$f(0) = 0 \Rightarrow \underline{W_2(0|0)}$

$f\left(\sqrt{\frac{3}{2}}\right) = 2\cdot\sqrt{\frac{3}{2}} - 2\cdot\sqrt{\frac{3}{2}}\cdot e^{1-\sqrt{3/2}^2} = \sqrt{6} - \sqrt{6}\cdot e^{-1/2} = \sqrt{6}\left(1 - \frac{1}{\sqrt{e}}\right)$

$\underline{W_3\left(\frac{1}{2}\sqrt{6} \,\middle|\, \sqrt{6}\left(1 - \frac{1}{\sqrt{e}}\right)\right)} \approx W_3(1{,}22 \,|\, 0{,}96)$; $\underline{W_1(\approx -1{,}22 \,|\, \approx -0{,}96)}$

(wegen Sym.!)